CP violation and Electroweak Baryogenesis in Extensions of the Standard Model

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ABSTRACT

We develop a new and general method to calculate the effects of CP violation from extensions of the standard model on the mechanism of electroweak baryogenesis. We illustrate its applicability in the framework of two-higgs doublet models.

1. Introduction

It has recently been convincingly established that electroweak baryogenesis^[1] due to the mixing of three generations of quarks in the minimal standard model is unable to account for today's baryon asymmetry.^[2,3,4] This result is the direct consequence of a sharp conflict between the rapid quark-gluon interactions and the far too slow processes of quantum interference through which the phase of the Kobayashi-Maskawa matrix can emerge into the physical world.^[4]

This result has long been anticipated and a plethora of new sources of CP violation have already been contemplated. There is a clear contrast between these new sources of CP violation and the standard model CKM phase. The latter is only physical because of the charge current interactions. In contrast, in many models with new sources of CP violation, during the weak phase transition some mass matrix has a space dependent phase which cannot be removed since making the masses real and diagonal at two adjacent points x and x + dx requires, in a space-varying background, two different unitary rotations U_x and U_{x+dx} . The relative rotation $U_x^{-1}U_{x+dx}$ yields a new interaction which can generate a CP violating observable. Because large physical CP violating interference effects can appear in the phase boundary where the particle masses are space dependent, they play a dominant role in the mechanism of electroweak baryogenesis.

At the electroweak phase transition, non equilibrium CP violating effects are largest inside the wall of a bubble of broken phase expanding in a thermally equilibrated plasma. Two apparently distinct mechanisms of electroweak baryogenesis have been proposed in the literature, the so-called "thin wall" and "thick wall" scenarios. They are characterized by the conditions $\ell/L \gg 1$ and $\ell/L \ll 1$ respectively, where L and ℓ are the wall thickness and a typical mean free path of the particles relevant to the scenario.

In the "thin wall" scenarios, coefficients of reflection and transmission are computed. For those species whose interactions with the wall take place through a complex mass matrix, these coefficients assume different values for a particle and its CP conjugate. The resulting asymmetry is then convoluted with an incoming thermal flux to yield a CP violating source which moves along with the interface. This method has been applied to exotic fermions with a majorana mass and to the top quark in two-higgs doublet models.^[6]

In the "thick wall" scenarios, thermally-averaged local operators have been written which couple the baryon current, or a related current, to the space-time derivative of the mass terms. These operators act as CP violating sources defined at every point of the interface. These are the scenarios proposed in the context of the two higgs^[7,8,9] and supersymmetric models.^[10,11]

It has recently been understood that in both scenarios these sources are to be inserted in a set of coupled rate equations which allow for the CP violating charges to be transported elsewhere in the plasma. Transport greatly enhances the final baryon asymmetry since anomalous electroweak baryon violating processes are suppressed in the wall and in the broken phase but are relatively rapid in the symmetric phase.^[12,13,14]

This dichotomy in the formulation of electroweak baryogenesis mechanisms obviously reflects limitations of the computational techniques and does not do justice to the underlying physics of CP violation. It is unsatisfactory for a few reasons.

- (a) In practice, the thickness of the wall is neither very small nor very large compared to a mean free path. The "thin wall" approximation can overestimate the magnitude of the actual baryon asymmetry produced while the "thick wall" approximation can underestimate it.
- (b) An interpolation between these two limits is required as the *CP* violating sources are the only inhomogeneous terms in the rate equations so that the uncertainty in the baryon asymmetry is directly proportional to the uncertainty in their determination.

- (c) For any given energy, particles moving at an oblique enough angle relative to the wall are likely to be scattered while inside the wall. Hence the integral over all particle momenta will include both particles which scatter inside the wall many times and those which do not scatter at all and the "thin wall" limit is never fully applicable.
- (d) It has previously been argued [7-11,13] that for a sufficiently thick wall it is a good approximation to compute the effects of the nonequilibrium CP violation by adding time varying CP and CPT violating terms to an effective Hamiltonian which is treated as approximately spatially constant, and by assuming that the local particle distributions relax towards thermal equilibrium with this Hamiltonian according to some classical rate equations. However, the plasma includes many particles with a small momentum perpendicular to the wall and so with a long wavelength perpendicular to the wall. When this wavelength is long compared with the particle mean free path, a "classical" treatment of the CP violation is not adequate.

What follows is the description of a new method of calculation which applies to all scenarios and all values of ℓ/L . It reflects in the most direct way the interplay between the coherent phenomenon of CP violation and the incoherent nature of the plasma physics. This method can account for the generation of a CP violating observable from mass matrices with non-trivial space-dependence, as well as from particle interactions. In its simplest form, it easily reproduces the "thin wall" and "thick wall" calculations wih significant improvements over earlier estimates. In its more general form, it can incorporate effects which arise from the large diversity of scales present in a realistic plasma and can be the basis for Monte-Carlo simulations. For reasons of clarity, the method is best introduced with an example: the two higgs doublet model. The reader should bear in mind that it applies to other theories as well.

^{*} The latter mechanism dominates when the former one is not present. This is the case in the minimal standard model with *CP* violation originating from the quark Yukawa couplings.^[4]

2. Construction of the CP violating sources

Let us consider a set of particles with (not necessarily diagonal) mass matrix M(z) and moving, in the rest frame of the wall, with energy-momentum E, \vec{k} . At their last scattering point z_o , these particles emerge from a thermal ensemble, propagate freely during a mean free time $\tau \sim \ell$, then rescatter and return to the local thermal ensemble in the plane $z_o + \tau v$, v being the velocity perpendicular to the wall, k_{\perp}/E . During the time τ , these particles evolve according to a set of Klein-Gordon, Dirac or Majorana equations coupled through the mass matrix M(z). It is in the course of this evolution that CP violation affects the distribution of these particles. Initially, at z_o , the contribution of these particles to any given charge cancel exactly the contribution of their antiparticles: $\langle Q \rangle = Tr[\hat{Q} - \hat{Q}] = 0$; here, \hat{Q} is the charge operator and the trace is taken over flavors as well as particle distributions. However, after evolving a time τ across the CP violating space-dependent background, this cancellation no longer takes place for those charges which are explicitly violated by the mass matrix M(z). At the subsequent scattering point $z_o + \tau v$, these charges become $\langle Q \rangle = Tr[A^{\dagger} \hat{Q} A - \overline{A}^{\dagger} \hat{Q} \overline{A}]$ and assume a non-zero value, as A, the evolution operator over the distance τv , is distinct from its CP conjugate A.

To be specific, let us define J_{\pm} , the average current resulting from particles moving toward positive(negative) z between z_o and $z_o + \Delta$, $\Delta = \tau v$. The current J_+ receives contributions from either particles originating from the thermal ensemble at point z_o , moving with a positive velocity and being transmitted at $z_o + \Delta$, or from particles originating at $z_o + \Delta$, moving with velocity -v and being reflected back towards $z_o + \Delta$ (Fig. 1a). A similar definition exists for J_- (Fig. 1b). J_{\pm} are CP violating currents which are associated with each layer of thickness Δ moving along with the wall. Once boosted to the plasma frame, these currents provide CP violating sources, which fuel electroweak baryogenesis.

The calculation of these currents is facilitated by the use of CPT symmetry and unitarity. CPT symmetry identifies the amplitude for a particle to be transmitted

from the left with the amplitude for its CP conjugate to be transmitted from the right, while unitarity relates transmission to reflection amplitudes. Instead of writing a cumbersome but general formula for these currents, let us work them out for a specific situation: a single fermion with a Dirac mass $M(z) = m(z)e^{i\theta(z)}$. It could be a top quark having its mass generated from a two higgs-doublet lagrangian with an explicit CP violating term in the higgs potential, in which case, τ is the mean free time for quark-gluon scatterings. As for the current, we choose the axial current † . For this situation, the four-vectors J_{\pm} take the form

$$J_{+}(z_{o}) = \int_{\tilde{v}>0} \frac{d^{3}\vec{k}}{(2\pi)^{3}} \left\langle \left[n(E,v) - n(E,-\tilde{v}) \right] Q(z_{o},\vec{k},\tau) \right\rangle_{z_{o}} (1,0,0,\tilde{v})$$

$$J_{-}(z_{o}) = \int_{\tilde{v}>0} \frac{d^{3}\vec{k}}{(2\pi)^{3}} \left\langle \left[n(E,v) - n(E,-\tilde{v}) \right] Q(z_{o},\vec{k},\tau) \right\rangle_{z_{o}} (1,0,0,-v) .$$
(1)

In this expression, $v, = k_{\perp}/E$, is the velocity perpendicular to the wall at point z_o , \tilde{v} is the velocity a distance Δ away, $\tilde{v}^2 = v^2 + (m^2(z_o) - m^2(z_o + \Delta))/E^2$ and n(E, v) is the Fermi-Dirac distribution n_f boosted to the rest frame of the wall, $= [\exp[\gamma_W E(1 - vv_W)] + 1]^{-1}$. $Q(z_o, \vec{k}, \tau)$ is the charge asymmetry which results from the propagation of particles of momentum \vec{k} in the interval $[z_o, z_o + \Delta]$. In our specific example, Q is the chiral charge and is given by

$$Q_A(z_o, \vec{k}, \tau) = |T_L|^2 - |T_R|^2 - |T_{\overline{L}}|^2 + |T_{\overline{R}}|^2, \tag{2}$$

where T_L is the amplitude for a left-handed spinor to propagate over the distance Δ , $T_R = T_L(M \to -M^{\dagger})$ and $T_{\overline{L}} = T_L(M \to M^*)$. Finally, the brackets $\langle \dots \rangle_{z_o}$ in Eq. (1), average the location of point z_o within a given layer of thickness Δ as scattering occurs anywhere within a layer.

[†] We choose this current because in the two Higgs model a combination of axial top number and Higgs number diffuses efficiently into the symmetric phase and is approximately conserved by scattering in the symmetric phase.^[8,13]

The standard "thin wall" and "thick wall" situations are obtained in taking τ/L to ∞ and 0, respectively. In the "thin wall" limit, $\tau/L \to \infty$, the amplitudes T become the usual transmission coefficients and our expressions (1) for J_{\pm} , match trivially earlier calculations of scattering of particles off a sharp interface. In the "thick wall" limit, $\tau/L \to 0$, the currents J_{\pm} yield, after a boost to the thermal frame, a locally defined space-time dependent source density S(x,t) which generalizes, and gives a precise meaning to, the local CP violating operators already considered in the literature. In our example, the source per unit volume per unit of time, located at a point x fixed in the plasma, at any given time t, is, to first order in v_W and τ/L ,

$$S_A(x,t) = \frac{-\gamma_w v_w}{2\pi^2} \int_0^1 dv \int_{z_o - \tau v/2}^{z_o + \tau v/2} \frac{dz}{\tau v} \int_{\gamma_m(z)}^{\infty} dE E^3 \left. \frac{dn_f}{dE} (2v) \left. \frac{Q_A(z, \vec{k}, \tau)}{\tau} \right|_{z_0 = \gamma_w(x - v_w t)}.$$
(3)

In order to obtain an explicit form for the source $S_A(x,t)$, we need to compute the CP violating charge Q_A and perform the integration in Eq. (3).

3. Computation of the charge asymmetry Q

In general, $Q(z_o, \vec{k}, \tau)$ is a charge asymmetry produced by particles moving with momentum \vec{k} between the planes z_o and $z_o + \tau v$. Its calculation may require a different technique depending on the relative values of the time scales involved and on the choice of the charge. The physics of the generation of a CP violating observable, is the physics of quantum interference. It is most easily dealt with by treating the mass M(z) as a small perturbation (M(z) < T). Using techniques developed in Ref. 4, one finds, for the transmitted amplitude,

$$T_{L}(z_{o},\tau) = e^{i\Delta k_{\perp}} \left[1 - \int_{z_{o}}^{z_{o}+\Delta} dz_{1} \int_{z_{o}}^{z_{1}} dz_{2} e^{i2k_{\perp}(z_{1}-z_{2})} M(z_{2}) M^{\dagger}(z_{1}) + \mathcal{O}(M/k_{\perp})^{4} \right].$$
(4)

This expression has a straightforward diagrammatic formulation presented in Fig.

2a. A similar expansion can be written for the reflection amplitude R_L (Fig. 2b). The various terms in the sum correspond to various paths with different CP odd and CP even phases. Only interference between these paths contributes to a CP violating physical observable such as $Q_A(z_o, \vec{k}, \tau)$; one finds

$$Q_{A}(z_{o}, \vec{k}, \tau) = 8 \int_{z_{o}}^{z_{o} + \Delta} dz_{1} \int_{z}^{z_{1}} dz_{2} \sin 2k_{\perp}(z_{1} - z_{2}) \operatorname{Im}[M(z_{2})M^{\dagger}(z_{1})] + \mathcal{O}(M/k_{\perp})^{4}.$$
(5)

This expression is valid for any wall size and shape, and generalizes easily to the case where many flavors mix. Given a wall profile, the integrals can be evaluated and Eq. (5) can be inserted in formulas (1) to provide an explicit form for the currents J_{\pm} .

It is simplest to work out the case of a very thick wall $L \gg \tau$. Using the derivative expansion $M(z_i) = M(z_o) + (z_o - z_i)\partial_z M(z_o)$, one finds

$$Q_A(z_o, \vec{k}, \tau) = -4f(k_{\perp} \Delta)/k_{\perp}^3 \operatorname{Im}[M^{\dagger} \partial_z M]_{z_o}$$

$$= -4f(k_{\perp} \Delta)/k_{\perp}^3 m^2 \partial_z \theta|_{z_o} \quad \text{with} \quad f(\xi) = \sin \xi \left(\sin \xi - \xi \cos \xi\right).$$
(6)

Inserting this latter expression into our formula (3) for the source density S_A yields

$$S_A(x,t) = -T \gamma_w v_w m^2 \partial_{z_o} \theta \big|_{z_o = \gamma_w(x - v_w t)} \times \frac{2}{\pi^2} \mathcal{I}(\tau, m, T) + \mathcal{O}\bigg(v_w^2, (m/T)^4, (\tau/L)^2\bigg).$$
(7)

 $\mathcal{I}(\tau,m,T)$ is a form factor whose general form is

$$\mathcal{I}(\tau, m, T) \simeq \frac{1}{\sqrt{\tau T}} \int_{\frac{\bar{\mathbf{m}}}{T}}^{\infty} dy \sqrt{y} \frac{e^{y}}{(e^{y} + 1)^{2}} \int_{0}^{\tau T(y - (\frac{\bar{\mathbf{m}}}{T})^{2} \frac{1}{y})} \frac{dt}{t^{3/2}} f(t)$$
with $\bar{\mathbf{m}}^{2} = m^{2} + M_{T}^{2}$. (8)

We have included thermal corrections, M_T , in the mass dependence of Eq. (8)to take into account modifications of particle dispersion relations from scattering. The

effects of scattering on particle propagation can be accounted for by substituting quasiparticles for particles, in which case, τ is to be replaced with $1/2\gamma$, where γ is the width of the quasiparticle. Correspondingly, the dispersion relation is to be modified to incorporate self-energy thermal corrections. In the particular case of quarks scattering off gluons, the width γ is $\simeq g_s^2 T/3$, while the main thermal corrections amount to the shift $E^2 \to E^2 + M_T^2$, with $M_T^2 = g_s^2 T^2/6 \simeq T^2/4$.

The form factor \mathcal{I} is plotted in Fig. 3. \mathcal{I} vanishes as $\tau \to 0$, it peaks at $\tau T \sim 1$ and is well approximated by $\sim 1/\sqrt{\tau T}$ in the range $\tau T > 5$. The interpretation of this behavior is straightforward. As explained earlier, constructive interference is maximal for particles whose transverse Compton wavelength k_{\perp}^{-1} is of the order of τ , that explains the peak at $\tau T \sim 1$. As τT increases, fast oscillations along the distinct paths tend to cancel against each other and the resulting asymmetry drops; as a matter of fact, in the extreme limit $\tau T \gg 1$, the propagation is semi-classical and the asymmetry vanishes as it should.

In the opposite limit, as $\tau T \to 0$, the asymmetry vanishes as the quantum coherence required is washed away by the rapid plasma interactions. Fig. 3b demonstrates the mild dependence of \mathcal{I} on $\bar{\mathbf{m}}$.

For the sake of comparison, we present an approximate form of Eq. (7), valid for $m \ll T$,

$$S_A(x,t) \simeq -\frac{1}{\pi^2} \frac{1}{\sqrt{\tau T}} \gamma_w v_w T \ m^2 \partial_z \theta \quad \text{for} \quad \tau T \ge 5$$
 (9)

$$S_A(x,t) \simeq -\frac{1}{2\pi^2} \gamma_w v_w T m^2 \partial_z \theta$$
 for $\tau T \sim 1 - 2$. (10)

^{*} A systematic method which accounts for the thermal structure of a quasiparticle in the interference mechanism is given in Ref. 4.

[†] However, a semi-classical treatment alone might miss the important contributions of long-wavelength particles moving at large angles in respect to the wall motion; without their contribution, the asymmetry would fall off as fast as $1/\tau$.

4. Application to two-higgs models

The formula we derived for the chiral source $S_A(x,t)$ can be directly applied to the top quark propagating in the thick wall of a bubble produced at the electroweak phase transition in two-higgs models. Here the mean free path is dominated by gluon scattering $\tau \sim 3/(2g_s^2T) \sim (1-2)/T$ and is typically smaller than the estimated thickness of the wall: $\tau/L \sim 0.01 - 0.1$. The wall velocity is $\gamma_w v_w < 1$. Finally, The mass of the top quark $m_t(z)$ is $Y_t \phi \leq Y_t T \simeq T$ while $M_T \simeq T/2$, hence, $\bar{\mathbf{m}} = \sqrt{m_t^2 + M_T^2} \leq 1.1T$ and the assumptions under which we derived Eq. (10) are approximately fulfilled. We find

$$S_A(x,t) \simeq -\frac{N_c}{2\pi^2} \gamma_w v_w T \ m^2 \partial_z \theta + \mathcal{O}\left(v_w^2, (\tau/L)^2, (m_t/T)^4\right),$$
 (11)

where the number of colors $N_c = 3$.

Recent work on the source terms for axial top number in the two Higgs model in the thick wall case, have treated the fermion interaction terms with the background Higgs field as a CP violating contribution to a classical Hamiltonian in computing the CP-violating perturbation to particle distributions. Ref. 8 pointed out that these interactions split the energy levels of particles and their CP conjugates in a way reminiscent of a chemical potential. These classical treatments obscure the origin of the CP violating effect as resulting from quantum interference. However, these methods, if implemented properly, should provide reasonable approximations to our formulae for those particles whose wavelength $1/k_{\perp}$ is short compared with $v\tau$. As an illustration, Ref. 13 found a source term $S_A(x,t) = 1/3 T v_w m^2 \partial_z \theta$. These authors did not account for the quark-gluon interactions in the rate for incoherent axial top number violation (a factor $1/\tau$) and the three dimensional phase space (a factor of $\sim 9\zeta(3)/\pi^2$), factors which are numerically unimportant but which are needed for theoretical self-consistency. Even after including these effects, our formulae do not agree for large τ because our integral (3) is dominated by particles with long wavelengths in the direction perpendicular to the wall, for which a classical approximation is not adequate. Numerically, for $\tau T \sim 2$, our answer approximately agrees in magnitude.

5. Conclusion

In conclusion, we have introduced a new method to compute CP violating sources resulting from particle interaction with an expanding bubble during a first-order electroweak phase transition.

- 1. The method refers to explicit physical processes in the plasma. In particular, it does not make use of thermally averaged operators, or effective chemical potentials whose connection to the microphysics is indirect, as they do not vanish as the mass m and/or the mean free time τ vanish, and whose applicability is restricted to the range $L\gg\tau\gg1/T$.
- 2. The method makes explicit the quantum physics of *CP* violation and its suppression resulting from thermal effects.
- 3. A major advantage of our formulation is that it easily applies to charges generated by flavor mixing through arbitrary large mass matrices. In particular, it can be applied to cases, such as the supersymmetric standard model [17] for which there is no known semi-classical approximation.
- 4. It is valid for all wall shapes and sizes as well as for arbitrary particle species and interactions.
- 5. Finally, it incorporates *CP* violation which originates from particle interactions as well as from non-trivial space-time mass dependences. In particular, it generalizes and agrees with the decoherence arguments invoked to rule out electroweak baryogenesis from *CP* violation in the quark Yukawa couplings, as given in Ref. 4.

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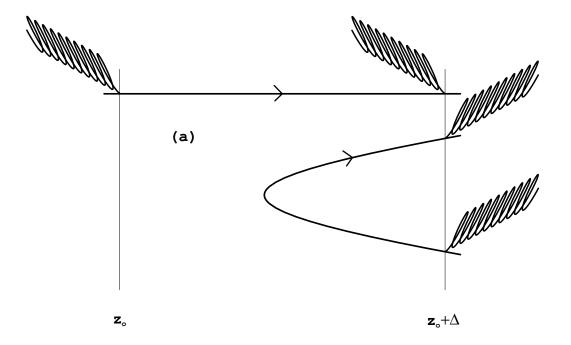
FIGURE CAPTIONS

- 1) (a) Contributions to J_+ . (b) Contributions to J_- .
- 2) (a) Amplitudes contributing to T_L . (b) Amplitudes contributing to R_L .
- 3) (a) The form factor \mathcal{I} is plotted versus τT , for the case $m \ll 0$, $M_T = T/2$ (quark-gluon interactions). The dotted curve results from numerical integration of Eq. (8); for values of $\tau T > a$ few, it is well-approximated by its asymptotic form $1/2\sqrt{\tau T}$ (solid line). (b) The dependence of \mathcal{I} on the mass $\bar{\mathbf{m}}$ is mild in the range $\bar{\mathbf{m}} \leq T$.

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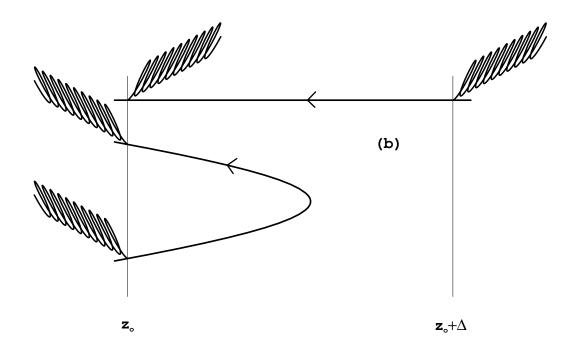
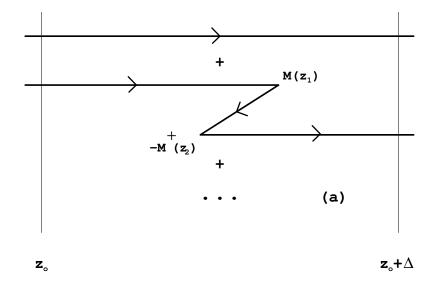


Fig. 1



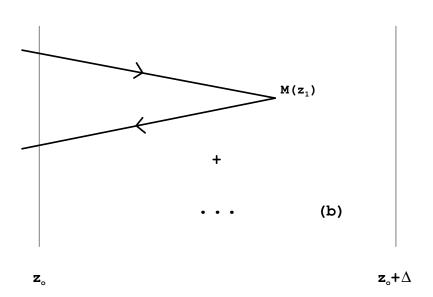
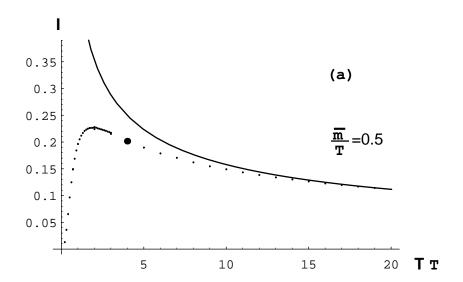
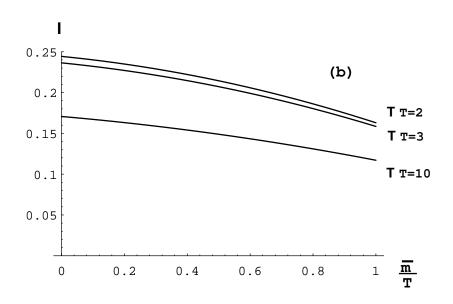


Fig. 2





¹⁶ **Fig. 3**